Seeking of an Optimal Path

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Abstract

Board of Secondary Education Karachi (BSEK) at Nazimabad No. 5, is the only Board that organizes examinations annually for IX and X class students of Arts, Commerce and Science groups. A number of students appeared in the said examinations from both private and public sector schools, who very frequently visit Board Office from different part of city to resolve their issues regarding enrolment, examination and certificate etc. However, the students residing in Korangi, Malir, DHA, Landhi, and adjoining areas are to travel a large distances and spend substantial time and money to reach BSEK. In order to facilitate and to address their problems, an effort is being made to find out a shortest path, that will provide ease and to alleviate difficulties faced by them. A mathematical model in this regard is being developed with the help of Floyd's algorithm to achieve the desired objectives.

Keywords

Floyd Algorithm, Iterations k = 0-7, Node 1-Korangi Area Karachi, Node 7-BSEK, Shortest Track.

1 Introduction

Board of Secondary and Higher Secondary Education Karachi was established in 1950, and started functioning in a three roomed residential house at Fatima Jinnah Colony, Jamshed Road, with a skeleton staff of 3 or 4 persons only. The newly constituted Board conducted its first SSC & HSC Examinations in May, 1962, for 17,000 and 8,000 examinees respectively. By this time it had 133 recognized schools, and 21 affiliated Intermediate colleges.The ordinance of 1972 was amended by an Act No. 23 of 1972 & No. 10 of 1974, whereby the said Board was bifurcated into two Boards, namely the Board of Secondary Education Karachi (BSEK) & the Board of Intermediate Education Karachi (BIEK). The BSEK remained in the old premises at Nazimabad No.5 Karachi whereas the BIEK shifted in the hostel building of National Youth Center. Mr. Rustom V. Divecha was appointed as first Chairman of the BSEK. The two Boards started functioning separately from 1st. February, 1974. (http://bsek.edu.pk-History of BSEK).

The population of Karachi is gradually increased and more private schools have been opened and registered by the Board. These private schools open with a commitment to provide better education facilities to the students, as such it is considered mandatory to give free hand to the philanthropist to open new schools in private sectors. As a result number of schools have been established and thereafter education is also become a profitable business.

Now a large number of students are visiting the BSEK Office for their miscellaneous problems from different parts of the city. In order to provide ease and to facilitate the students / teachers and other peoples, an effort has been made to find out an optimal path, from Korangi to the BSEK at Nazimabad No. 5.

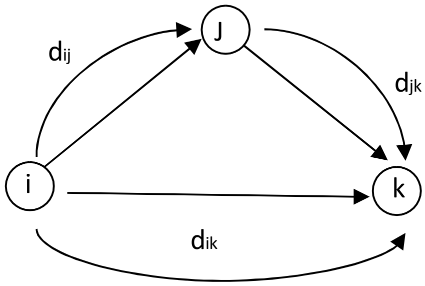
To achieve the objective, internet site of google map (https://maps.google.com/maps/), the city government, Highway Authority and concerned department have been approached to get desired information.

To find the shortest path from Korangi to BSEK the Floyd Algorithm is considered appropriate for the purpose. The algorithm is designed to calculate the shortest path between the source node and every other node, moreover it also provide the shortest route between any two nodes in the same network. The algorithm represent n-nodes network as a square matrix having n rows and n column entry (i,j).

2 Methodology

In the present case the Floyd's Algorithm is considered suitable for finding the shortest path, it is an n-nodes network as square matrix with n-rows and n-Columns entry (i,j), however d_{ij} gives the distance from the node i to j which is finite if i is linked directly to j and infinite otherwise. Given three nodes i, j and k as shown in Figure 1 with the connecting distances shown by the three arcs, it is shorter to reach k from i passing through j if $d_{ij} + d_{jk} < d_{ik}$

Fig. 1



It is optimal to replace the direct route from i ♉ k with the indirect route i ♉ j ♉ k. This triple operation is applied symmetrically to the network using the following steps.

Step 0: Defining the starting distance matrix D_0 and node sequence matrix S_0. The diagonal elements are marked with (--) to indicate that they are blocked. Set k = 1

Step k: Define the row k and column k as pivot row and pivot column. Apply the triple operation each d_{ij} in D_{k-1} for all i and j. if the condition $d_{ik} + d_{kj} < d_{ij}$,$(i \neq k , j \neq k)$ is satisfied , make the following changes .

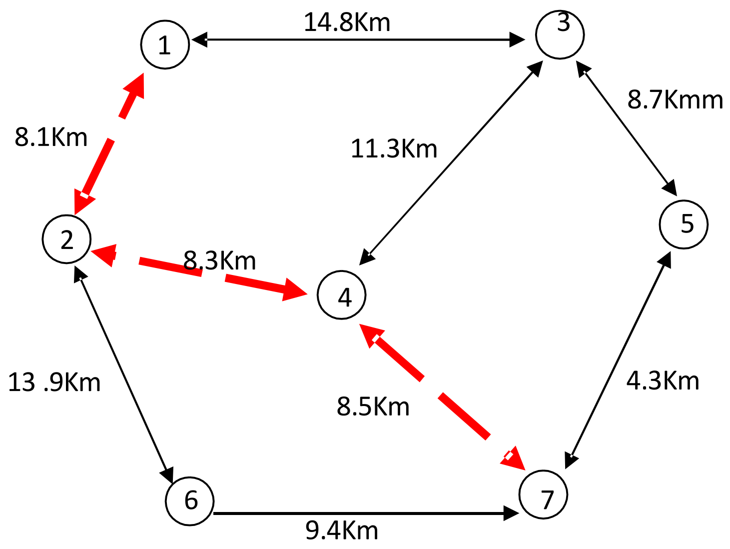
(a) Create D_k by replacing d_{ij} in D_{k-1} with $d_{ik} + d_{kj}$.
(b) Create S_k by replacing S_{ij} in S_{k-1} with k Set
k = k + 1 and repeat step k.

After 1, 2, 3 … n, step we can determine the shortest route/ path between node i and j from the matrices D_n and S_n using the following,

(1) From D_n ,d_{ij} give the shortest distance between the node i and j .
(2) From S_n, determine the intermediate node $k = S_{ij}$, which yields the route
i ♉ k ♉ j,

If S_{ik} = k and S_{kj} = j stop, all the intermediate nodes of the route have been found otherwise repeat the procedure between the node i and k, and node k and j [1-5]. For the present scenario the network diagram is shown in Fig. 2

Fig. 2



Node 1	Korangi Area Karachi
Node 2	Natha Khan Shara-e-faisal
Node 3	FTC Shara-e-faisal
Node 4	Hassan Square Gulshan-e-Iqbal
Node 5	LasbelaChowrangi
Node 6	NagunChowrangi
Node 7	BSEK at Nazimabad No. 5

Iteration k = 0

The matrices D_0 and S_0 give the initial representation of the network, D_0 is symmetric except that $d_{76} = \infty$ because no traffic is allowed from node 7 to node 6.

D_0

	1	2	3	4	5	6	7
1	--	8.1	14.	∞	∞	∞	∞
2	8.1	--	∞	8.3	∞	13.	∞
3	14.	∞	--	11.	8.	∞	∞
4	∞	8.3	11.	--	∞	∞	8.
5	∞	∞	8.7	∞	--	∞	4.
6	∞	13.	∞	∞	∞	--	9.
7	∞	∞	∞	8.5	4.	∞	--

S_0

	1	2	3	4	5	6	7
1	-	2	3	4	5	6	7
2	1	-	3	4	5	6	7
3	1	2	-	4	5	6	7
4	1	2	3	-	5	6	7
5	1	2	3	4	-	6	7
6	1	2	3	4	5	-	7
7	1	2	3	4	5	6	-

Iteration k = 1

The pivot row and pivot column are given by the first row and first column (k = 1), as Shown in the D_0 Matrix the element d_{23} and d_{32} can be written by the triple operation thus to obtain D_1 and S_1 from D_0 and S_o

Replace $d_{23} = d_{21} + d_{13} = 8.1 + 14.8 = 22.9 = d_{32}$ $S_{23} = 1$ and $S_{32} = 1$

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D_1

S_1

	1	2	3	4	5	6	7
1	--	8.1	14.8	∞	∞	∞	∞
2	8.1	--	22.9	8.3	∞	13.9	∞
3	14.8	22.9	--	11.3	8.7	∞	∞
4	∞	8.3	11.3	--	∞	∞	8.5
5	∞	∞	8.7	∞	--	∞	4.3
6	∞	13.9	∞	∞	∞	--	9.4
7	∞	∞	∞	8.5	4.3	∞	--

	1	2	3	4	5	6	7
1	-	2	3	4	5	6	7
2	1	-	1	4	5	6	7
3	1	1	-	4	5	6	7
4	1	2	3	-	5	6	7
5	1	2	3	4	-	6	7
6	1	2	3	4	5	-	7
7	1	2	3	4	5	6	-

Iteration k = 2

The triple operation is applied in D_1 and S_1 the changes are shown in D_2 and S_2

Replace d_{14} with $d_{14} = d_{12} + d_{24} = 8.1 + 8.3 = 16.4 = d_{41}$; set $S_{14} = 2$ and $S_{41} = 2$ similarly

Replace $d_{36} = d_{32} + d_{26} = 22.9 + 13.9 = 36.8 = d_{63}$ $S_{36} = 2$ and $S_{63} = 2$

D_2

	1	2	3	4	5	6	7
1	--	8.1	14.	16.4	∞	22	∞
2	8.1	--	22.	8.3	∞	13.	∞
3	14.	22.	--	11.	8.	36.8	∞
4	16.4	8.3	11.	--	∞	∞	8.
5	∞	∞	8.7	∞	--	∞	4.
6	22	13.	36.8	∞	∞	--	9.
7	∞	∞	∞	8.5	4.	∞	--

S_2

	1	2	3	4	5	6	7
1	-	2	3	2	5	2	7
2	1	-	1	4	5	6	7
3	1	1	-	4	5	2	7
4	2	2	3	-	5	6	7
5	1	2	3	4	-	6	7
6	2	2	2	4	5	-	7
7	1	2	3	4	5	6	-

Iteration k = 3

The triple operation is applied in D_2 and S_2 the changes are shown in D_3 and S_3

Replace $d_{15} = d_{13} + d_{35} = 14.8 + 8.7 = 23.5 = d_{51}$ and set $S_{15} = 3$

Replace $d_{51} = d_{53} + d_{31} = 8.7 + 14.8 = 23.5 = d_{51}$ and set $S_{51} = 3$
Similarly,

Replace $d_{25} = d_{23} + d_{35} = 22.9 + 8.7 = 31.6 = d_{52}$ $S_{25} = 3$ and $S_{52} = 3$

D_3

	1	2	3	4	5	6	7
1	--	8.1	14.8	16.4	23.5	22	∞
2	8.1	--	22.9	8.3	31.6	13.9	∞
3	14.8	22.9	--	11.3	8.7	36.8	∞
4	16.4	8.3	11.3	--	∞	∞	8.5
5	23.5	31.6	8.7	∞	--	∞	4.3
6	22	13.9	36.8	∞	∞	--	9.4
7	∞	∞	∞	8.5	4.3	∞	--

S_3

	1	2	3	4	5	6	7
1	-	2	3	2	3	2	7
2	1	-	1	4	3	6	7
3	1	1	-	4	5	2	7
4	2	2	3	-	5	6	7
5	3	3	3	4	-	6	7
6	2	2	2	4	5	-	7
7	1	2	3	4	5	6	-

Iteration k = 4

The triple operation is applied in D_3 and S_3 the changes are shown in D_4 and S_4
Replace $d_{23} = d_{24} + d_{43} = 8.3 + 11.3 = 19.6 = d_{32}$ $S_{23} = 4$ and $S_{32} = 4$ So, d_{25} and d_{36} also improve
Replace $d_{25} = d_{27} + d_{75} = 16.8 + 4.3 = 21.1 = d_{52}$ $S_{25} = 7$ and $S_{52} = 7$
Replace $d_{36} = d_{34} + d_{46} = 11.3 + 22.2 = 33.5 = d_{63}$ $S_{36} = 4$ and $S_{63} = 4$ and
Replace $d_{17} = d_{14} + d_{47} = 16.4 + 8.5 = 24.9 = d_{71}$ $S_{17} = 4$ and $S_{71} = 4$
Replace $d_{27} = d_{24} + d_{47} = 8.3 + 8.5 = 16.8 = d_{72}$ $S_{27} = 4$ and $S_{72} = 4$
Replace $d_{37} = d_{35} + d_{57} = 8.7 + 4.3 = 13 = d_{73}$ $S_{37} = 5$ and $S_{73} = 5$

Replace $d_{76} = d_{74} + d_{46} = 8.5 + 22.2 = 30.7 \neq d_{67} = 9.4$ shows one way route $S_{76} = 4$

D4

	1	2	3	4	5	6	7
1	--	8.1	14.	16.4	23.	22	24.
2	8.1	--	19.	8.3	21.	13.	16.
3	14.	19.	--	11.	8.7	33.	13
4	16.4	8.3	11.	--	∞	∞	8.5
5	23.	21.	8.7	∞	--	∞	4.3
6	22	13.	33.5	∞	∞	--	9.4
7	24.	16.	13	8.5	4.3	30.	--

S4

	1	2	3	4	5	6	7
1	-	2	3	2	3	2	4
2	1	-	4	4	7	6	4
3	1	4	-	4	5	4	5
4	2	2	3	-	5	6	7
5	3	7	7	4	-	6	7
6	2	2	4	4	5	-	7
7	4	4	5	4	5	4	-

Iteration k = 5

The triple operation is applied in D_4 and S_4 the changes are shown in D_5 and S_5

Replace $d_{46} = d_{42} + d_{26} = 8.3 + 13.9 = 22.2 = d_{64}$ $S_{46} = 2$ and $S_{64} = 2$ also

Replace $d_{64} = d_{67} + d_{74} = 9.4 + 8.5 = 17.9 \neq d_{46}$ $S_{64} = 7$

D5

	1	2	3	4	5	6	7
1	--	8.1	14.8	16.4	23.5	22	24.9
2	8.1	--	19.6	8.3	28.3	13.9	16.8
3	14.8	19.6	--	11.3	8.7	33.5	13
4	16.4	8.3	11.3	--	∞	22.2	8.5
5	23.5	28.3	8.7	∞	--	∞	4.3
6	22	13.9	33.5	17.9	∞	--	9.4
7	24.9	16.8	13	8.5	4.3	30.7	--

S5

	1	2	3	4	5	6	7
1	-	2	3	2	3	2	4
2	1	-	4	4	7	6	4
3	1	4	-	4	5	4	5
4	2	2	3	-	5	2	7
5	3	7	3	4	-	6	7
6	2	2	4	7	5	-	7
7	4	4	5	4	5	4	-

Iteration k = 6

The triple operation is applied in D_5 and S_5 the changes are shown in D_6 and S_6

Replace $d_{45} = d_{43} + d_{35} = 11.3 + 8.7 = 20 = d_{54}$ $S_{45} = 3$ and $S_{54} = 3$

Replace $d_{45} = d_{47} + d_{75} = 8.5 + 4.3 = 12.8 = d_{54}$ $S_{45} = 7$ and $S_{54} = 7$

$d_{45} = 12.8$ and 20 so disregard the higher distance i.e 20 Km

D_6

	1	2	3	4	5	6	7
1	--	8.1	14.	16.4	23.	22	24.
2	8.1	--	19.	8.3	21.	13.	16.
3	14.	19.	--	11.	8.7	33.	13
4	16.4	8.3	11.	--	12.	22.	8.5
5	23.	21.	8.7	12.	--	∞	4.3
6	22	13.	33.	17.	∞	--	9.4
7	24.	16.	13	8.5	4.3	30.	--

S_6

	1	2	3	4	5	6	7
1	-	2	3	2	3	2	4
2	1	-	4	4	7	6	4
3	1	4	-	4	5	4	5
4	2	2	3	-	7	2	7
5	3	7	3	7	-	7	7
6	2	2	4	7	5	-	7
7	4	4	5	4	5	4	-

Iteration k = 7

The triple operation is applied in D_6 and S_6 the changes are shown in D_7 and S_7

Replace $d_{36} = d_{34} + d_{46} = 11.3 + 22.2 = 33.5$ $S_{36} = 4$

Replace $d_{36} = d_{31} + d_{16} = 14.8 + 22 = 36.8$ $S_{36} = 1$

$d_{36} = 33.5$ and 36.8, so disregard the higher distance i.e 36.8 Km

Replace $d_{63} = d_{67} + d_{73} = 9.4 + 13 = 22.4$ $S_{63} = 7$

Replace $d_{63} = d_{64} + d_{43} = 17.9 + 11.3 = 29.2$ $S_{63} = 4$

$d_{63} = 22.4$ and 29.2, so disregard the higher distance i.e 29.2 Km

Replace $d_{56} = d_{57} + d_{76} = 4.3 + 30.7 = 35 \neq d_{65} = 13.7$ shows one way route $S_{65} = 7$

Replace $d_{65} = d_{67} + d_{75} = 9.4 + 4.3 = 13.7$ $S_{65} = 7$

D7 ` S7

	1	2	3	4	5	6	7
1	--	8.1	14.	16.4	23.	22	24.
2	8.1	--	19.	8.3	21.	13.	16.
3	14.	19.	--	11.	8.7	33.	13
4	16.4	8.3	11.	--	12.	22.	8.5
5	23.	21.	8.7	12.	--	35	4.3
6	22	13.	22.	17.	13.	--	9.4
7	24.9	16.8	13	8.5	4.3	30.	--

	1	2	3	4	5	6	7
1	-	2	3	2	3	2	4
2	1	-	4	4	7	6	4
3	1	4	-	4	5	4	5
4	2	2	3	-	7	2	7
5	3	7	3	7	-	7	7
6	2	2	7	7	7	-	7
7	4	4	5	4	5	4	-

3 Results and Discussion

The shortest route that is optimal path from node 1 (Korangi) to node 7 (BSEK) has been calculated with the help of Floyd's algorithm and is equal to 24.9 km, the same is evident from iteration 7 which is appended below:

D7 S7

	1	2	3	4	5	6	7
1	--	8.1	14.		23.	22	24.
2	8.1	--	19.	8.3	21.	13.	16.
3	14.	19.	--	11.	8.7	33.	13
4		8.3	11.	--	12.	22.	8.5
5	23.	21.	8.7	12.	--	35	4.3
6	22	13.	22.	17.	13.	--	9.4
7				8.5	4.3	30.	--
	24.9	16.8	13				

	1	2	3	4	5	6	7
1	-	2	3	2	3	2	4
2	1	-	4	4	7	6	4
3	1	4	-	4	5	4	5
4	2	2	3	-	7	2	7
5	3	7	3	7	-	7	7
6	2	2	7	7	7	-	7
7	4	4	5	4	5	4	-

The final matrices D_7 and S_7 contain all the information needed to determine the shortest route between any two nodes in the network.

Therefore the shortest path from Korangi (Node 1) to BSEK (Node 7) is d_{17} = 24.9 km .To determine associated route, consider the segment (i,j) that represents a direct link If $S_{ij} = j$

But in S_7 $S_{17} = 4 \neq 7$ this shows that there is an intermediate node between 1- 7 that is 4 that links node 1 to node 7 that is 1 4 7

Now Similarly $S_{14} = 2 \neq 4$ therefore, the segment 1 - 4 is not directly linked.

Hence there is another intermediate node between 1- 4 i.e. 2

The route 1 4 is replaced by 1 2 4

$S_{12} = 2$, $S_{24} = 4$, $S_{47} = 7$, no further intermediate nodes exist

So the route from node 1(Korangi) to node 7 (BSEK) is

1 2 4 7 and the shortest path obtained is d_{17} = 24.9 Km

4 Conclusion:

In order to facilitate the students, teachers and other peoples of Karachi, a shortest route between Korangi Area Karachi and BSEK has been calculated, which is d_{17} = 24.9 km. However the route includes, the node points 1,2,4,7 (i.e) students or teachers coming from Korangi to BSEK should follow the route as under:

Korangi♉Natha Khan Shara-e-Faisal ♉Hassan Square♉BSEK. The route is said to be the shortest path and distance covered by the individuals will be 24.9 Km.

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